

Implications of a constant observed braking index for young pulsar's spindown

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ABSTRACT

The observed braking index n_{obs} which had been determined for a few young pulsars, had been found to differ from the expected value for a rotating magnetic dipole model. In addition, the observational jerk parameter, determined for two of these pulsars, disagrees with the theoretical prediction $m_{obs} = 15$ in both cases. We propose a simple model able to account for these differences, based on a growth of the "torque function" $K = -\dot{\Omega}/\Omega^n$, under the constraint that n_{obs} is a constant. We show that there is observational evidence supporting the latter hypothesis, and derive initial values for several physical quantities for the four pulsars whose n_{obs} have been measured.

Subject headings: Pulsars : general — Pulsars : individual : (PSR B0531+21, PSR B0540-69, PSR B0833-45, PSR B1509-58)

1. Introduction

The well-known vacuum dipole model expresses the external torque acting on pulsars as

$$I \dot{\Omega} = -\frac{2}{3c^3} B^2 R^6 \sin^2 \alpha \Omega^n \quad (1)$$

where α is the angle between B and Ω , B and R are the magnetic field and the radius of the star respectively, I is the star's moment of inertia and c is the velocity of light. The

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braking index n is usually assumed to be 3 as predicted by the magnetic dipole model (see e.g. Manchester & Taylor 1977), but actually the four (young) pulsars with the best determinations of braking index show values < 3 (see Blandford & Romani 1988 and references therein). In the general literature all these quantities are taken to be constants (except of course for Ω and its derivatives), which allows one to rewrite eq.(1) as

$$\dot{\Omega} = -K \Omega^n \quad (2)$$

where K is a function that absorbs the structural factors, and we will refer to throughout this work as the "torque function".

There is evidence that in 1975 and 1989 the spin rate Ω of the Crab pulsar suddenly increased by amounts $\Delta\Omega/\Omega \sim 10^{-8}$ and after that continued to spin-down at a faster rate (that is, the pulsar continued to spin *slower* than before) of $\Delta\dot{\Omega}/\dot{\Omega} \sim 10^{-4}$ (Gullahorn et al. 1977, Lohsen 1981, Lyne, Smith & Pritchard 1992), characterizing a *permanent* deficit in $\dot{\Omega}$. The same feature is also present in the 1969, 1981 and 1986 glitches (Lyne & Pritchard 1987, Lyne, Pritchard & Smith 1993). From eq.(1) it is clear that the peculiar events of 1975 and 1989 require either a reduction of I (Alpar & Pines 1993), an increase of the magnitude of the magnetic field B (Blandford, Applegate & Hernquist 1983, Muslimov & Page 1996, Camilo 1997) or an increase of the angle α (Macy 1974). Other dynamical models in which the angle between the magnetic dipole and the rotation axis of the pulsar is allowed to vary have been proposed by Link & Epstein (1997) and Allen & Horvath (1997). In our previous work, we assumed several laws for the growth of the angle α and verified which ones provided consistent solutions. It was shown that an exponential angular growth with a e -folding time of $\sim 10^4$ yr could fit the small braking indexes and jerk parameters of the Crab, Vela and B0540-69 pulsars, and although PSR B1509-58 was best-fitted by a logarithmic law, an exponential one is not ruled out because of the large uncertainties (25 %) on its jerk value.

The key feature here is the variation of the torque function K , which cannot be held constant, but instead evolves, growing on timescales shorter than the pulsar's life span. This work presents a simple general model for torque function growth, based on the observational feature $n_{obs} = \text{constant}$ (see also Ruderman 1993 and references therein). In the next Sections we will show that this simple hypothesis is enough to determine the general behaviour of the pulsar dynamics, along with its evolution on the $\dot{P} \times P$ diagram, yet taking into account the low braking indexes displayed by young pulsars.

2. Consequences of $n_{obs} = \text{constant}$

2.1. General model

To relate the model to the observations, we need to define some physical quantities. The first one is the variation of the torque function ΔK due to persistent shifts $\Delta\dot{\Omega}/\dot{\Omega}$ and $\Delta\Omega/\Omega$, which is easily found from eq.(2) to be

$$\Delta K = \left(\frac{\Delta\dot{\Omega}}{\dot{\Omega}} - 3 \frac{\Delta\Omega}{\Omega} \right) K. \quad (3)$$

Dividing eq.(3) by Δt , here defined as a typical timescale between glitches, we obtain a mean increase rate which we shall denote as $\langle \frac{\Delta K}{\Delta t} \rangle$. Note that even the exact definition of the mean is not completely satisfactory given the different coverages and observational biases for each pulsar (we will return to this point below). Using the data from Lyne, Pritchard & Smith (1993) (hereafter LPS), it can be found that the mean torque function variation caused by glitches (*discrete* variation) for the Crab is

$$\left\langle \frac{\Delta K}{\Delta t} \frac{1}{K} \right\rangle \simeq 3 \times 10^{-5} \text{ yr}^{-1} \quad (4)$$

with $\Delta t = 4.6 \text{ yr}$.

It can be shown that even if the power of Ω in the torque expression is exactly 3, the observed braking index defined as $n_{obs} = \ddot{\Omega}\Omega/\dot{\Omega}^2$ is

$$n_{obs} = 3 + \frac{\Omega}{\dot{\Omega}} \frac{\dot{K}}{K}, \quad (5)$$

and the observed jerk parameter defined as $m_{obs} = \ddot{\Omega}\Omega^2/\dot{\Omega}^3$ is

$$m_{obs} = 3(3n_{obs} - 4) + (n_{obs} - 3)^2 \left(\frac{\ddot{K}}{\dot{K}} \frac{K}{K^2} \right). \quad (6)$$

Eq.(5) can be inverted to find the (*continuous*) torque function variation needed to account for the observed braking index of Crab

$$\frac{\dot{K}}{K} = (n_{obs} - 3) \frac{\dot{\Omega}}{\Omega} \simeq 1.9 \times 10^{-4} \text{ yr}^{-1}. \quad (7)$$

So, the discrete contribution is only $\sim 15\%$ of the continuous torque function variation. That means that even pulsars that have not shown any glitches could have their torque functions varying in a continuous way, and also satisfy $n_{obs} < 3$. In fact, PSR B1509-58

and PSR B0540-69 have never displayed glitches, yet their braking indexes are 2.8 and 2.0 respectively, although the hypothesis that they are actually glitching at a rate comparable to the Crab is not ruled out. Application of eq.(7) to Vela, PSR B1509-58 and PSR B0540-69 yields 6.98×10^{-5} , 5.25×10^{-5} and $28.8 \times 10^{-5} \text{ yr}^{-1}$, respectively.

With the expression for n_{obs} we derive a form for the jump of the latter in each glitch event, namely

$$\Delta n_{obs} = n_{obs} \left(\frac{\Delta \Omega}{\Omega} - 2 \frac{\Delta \dot{\Omega}}{\dot{\Omega}} + \frac{\Delta \ddot{\Omega}}{\ddot{\Omega}} \right). \quad (8)$$

The division of eq.(8) by Δt yields a mean variation rate of n_{obs} , which can be calculated from LPS for the Crab to be

$$\left\langle \frac{\Delta n_{obs}}{\Delta t} \right\rangle \simeq -1.5 \times 10^{-4} \text{ yr}^{-1} \quad (9)$$

assuming that $\ddot{\Omega}$ did not vary in the events, as assumed in all data analysis performed until now. According to LPS the data show that n_{obs} appears to be constant within 0.5% (~ 0.01) over 20 years, implying $|\dot{n}_{obs}| < 5 \times 10^{-4} \text{ yr}^{-1}$, which is consistent with eq.(9). Thus we are led to explore a scenario in which pulsars evolve along a constant braking index value $\neq 3$.

A direct measurement of \dot{n}_{obs} in term of observables can be obtained as

$$\dot{n}_{obs} = n_{obs} \left(\frac{\ddot{\Omega}}{\dot{\Omega}} + \frac{\dot{\Omega}}{\Omega} - 2 \frac{\ddot{\Omega}}{\dot{\Omega}} \right) = \frac{m_{obs} - n_{obs}(2n_{obs} - 1)}{\Omega/\dot{\Omega}}. \quad (10)$$

Setting $\dot{n}_{obs} = 0$, we can express m_{obs} as function of n_{obs} , $\dot{\Omega}$ and Ω , and replace it in eq.(6), yielding

$$\frac{\ddot{K} K}{\dot{K}^2} = 1 - \frac{n_{obs} - 1}{3 - n_{obs}} = \text{constant} \quad (11)$$

which can be now integrated from an arbitrary point to the present values (indicated by the subscript p), resulting in

$$\dot{K} = \dot{K}_p \left(\frac{K}{K_p} \right)^{1 - \frac{n_{obs}-1}{3-n_{obs}}}. \quad (12)$$

Eq.(5) can be also integrated in the same way, and we obtain

$$\left(\frac{\Omega}{\Omega_p}\right)^{n_{obs}-3} = \frac{K}{K_p}. \quad (13)$$

Again, integration is performed and with the help of eq.(7) the time dependence of K is found to be (for $n_{obs} \neq 1$)

$$\frac{K}{K_p} = \left[1 + (n_{obs} - 1)(t_p - t)\frac{\dot{\Omega}_p}{\Omega_p}\right]^{\frac{3-n_{obs}}{n_{obs}-1}} \quad (14)$$

where t_p is the true age of the pulsar; the time dependence of Ω is trivially recovered substituting eq.(14) into eq.(13)

$$\frac{\Omega}{\Omega_p} = \left[1 + (n_{obs} - 1)(t_p - t)\frac{\dot{\Omega}_p}{\Omega_p}\right]^{\frac{1}{1-n_{obs}}}. \quad (15)$$

It is worth remarking that eq.(15) has the same form of the standard calculation made by assuming that the torque function does not change, with n_{obs} replacing $n = 3$ (see Manchester & Taylor 1977). In other words, it appears as though the actual external torque acting on pulsars has the form $\dot{\Omega} \propto \Omega^{n_{obs}}$. Therefore, several approximations customarily made in the literature are exact in the limit $n_{obs} = \text{constant}$.

Finally, from eq.(10) we write

$$m_{obs} = n_{obs}(2n_{obs} - 1). \quad (16)$$

Again, we note that eq.(16) has the same form as the conventional relation, if we replace $n = 3$ by n_{obs} .

It is important to note that the lower n_{obs} is, the closer the initial period is to the present one. Therefore, the use of the characteristic age $\tau = \frac{1}{(n_{obs}-1)} \frac{\Omega}{|\dot{\Omega}|}$ (obtained from eq.(15)), though strictly valid when $n_{obs} = \text{constant}$, introduces a non-negligible error because $\Omega \simeq \Omega_o$.

The important fact is that postulating $\dot{n}_{obs} = 0$ (as suggested by observations) completely determines the dynamical evolution of the pulsar, whichever model of variation one chooses between magnetic field, moment of inertia or angle α . In Table 1 we show the correspondence between K and its derivatives for all specific models.

In Fig.1 we depict the evolutionary tracks of the Crab and B1509-58 pulsars predicted by our model, compared to the standard model tracks, from birth to 5×10^4 years. Fig.2

shows the same for Vela and B0540-69. Remarkably, \dot{P} for Vela is actually increasing with time, although this does not mean a rising torque, because of the compensating effect of the power of Ω .

Now, it is easy to calculate the initial period for any pulsar whose n_{obs} has already been determined. In Table 2 we show observed and calculated quantities for the above pulsars, where $L = I\Omega\dot{\Omega}$ is the energy-loss rate of the rotating magnetic dipole. The data used in calculations came from LPS (Crab), Kaspi et al.(1994) (PSR B1509-58), Taylor et al. (1995, unpublished, see Taylor, Manchester & Lyne 1993) (PSR B0540-69), Lyne et al. (1996) and Taylor, Manchester & Lyne (1993) (Vela). Except in the case of Crab, the conventional characteristic age $\Omega/(2\dot{\Omega})$ was arbitrarily used as the true age of the pulsar, lacking a more reliable determination.

2.2. Crab Pulsar (PSR B0531+21)

The Crab pulsar is known to be among the most active glitching pulsars. If the glitch activity has been approximately the same since its birth, n_{obs} would have decreased by an amount $\langle \Delta n_{obs} / \Delta t \rangle \times t_p \sim 0.15$, or 6% of the present value. In the 25 years that have passed since the first Crab measurements, glitches could have provoked a reduction of only 0.004 (0.2%) in n_{obs} , far below the observational limit (see LPS). About 20 years' more data will be needed to unambiguously detect n_{obs} variation.

The values obtained assuming $n_{obs} = \text{constant}$ (Table 2) are very close to those observed, which is as expected, since $\dot{\Omega}$ was in fact estimated by LPS using eq.(16). The good fit to the data reinforces that \dot{n}_{obs} is zero or very small.

2.3. Vela Pulsar (PSR B0833-45)

Some authors (Aschenbach, Egger & Trümper 1995, Lyne et al. 1996) have discussed the possibility of Vela being up to 3 times older than its conventional characteristic age. As a test of the sensitivity of this model to the true age adopted, an age doubling alters the values in Table 2, as follows

$$P_o \simeq 26 \text{ ms}$$

$$K_o \simeq 1.56 \times 10^{-15} \text{ s} \tag{17}$$

$$\dot{P}_o \simeq 60.4 \times 10^{-15}$$

$$\frac{L_o}{L_p} \simeq 20$$

It is worth to noting that Vela is also one of the most glitch-active pulsars known, and, like the Crab, there could be a discrete torque function growth, as discussed in the introduction, which would make $\dot{n}_{obs} \neq 0$. However, timing analyses made to date (Lyne et al. 1996 being the most recent) have failed to unambiguously detect *permanent* shifts in $\dot{\Omega}$, because the typical (non-permanent) shift $\Delta\dot{\Omega}/\dot{\Omega}$ is 10^{-3} and the mean time between glitches is ~ 2 yr, leading to the conclusion that if such a permanent component does exist, it must be negligible.

3. Conclusions

We have shown that low braking indices, observed in young pulsars, can be attributed to a variation of the torque function K . Besides, there are evidence that $n_{obs} = \text{constant}$ in some of these pulsars, and this hypothesis leads to a complete determination of their dynamics, which we have calculated. Our expressions are almost identical to the conventional ones, when n substitutes n_{obs} , justifying the customarily estimates.

From Table 2, we can see that K must have increased by 25% to 100% since the pulsar birth. From the equivalence in Table 1, we verify that this means a reduction of 20-50% in the moment of inertia, which can be accomodated neither in the current vortex creep model nor as loss of oblateness. So a decrease in the moment of inertia cannot be invoked as the main physical agent behind torque evolution, although it probably is in the case of glitches.

An inspection of Table 2, Fig.1 and Fig.2 reveals sistematically larger initial periods and smaller period derivatives than those obtained in the standard model. It becomes clear that, within $n_{obs} = \text{constant}$ models, the characteristic age is poorly related to the true age of pulsars below 10^5 yr. This may affect statistical studies of pulsar populations.

We think this torque evolution, especially if driven by angular growth, probably stops at ages of the order of 10^4 years, when $\alpha = 90^\circ$ (Allen & Horvath 1997). Further improvement will require observations of the braking indices of middle-aged ($\sim 10^5$ years) pulsars.

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Table 1. Equivalence between expressions of general and specific models

General	Angle	Magnetic Field	Moment of Inertia
K	$\sin^2 \alpha$	B^2	I^{-1}
\dot{K}/K	$2\dot{\alpha}/\tan \alpha$	$2\dot{B}/B$	$-\dot{I}/I$
\ddot{K}/\dot{K}	$2 \cos(2\alpha)\dot{\alpha} + (\ddot{\alpha}/\dot{\alpha})$	$(\dot{B}/B) + (\ddot{B}/\dot{B})$	$(-2\dot{I}/I) - (\ddot{I}/\dot{I})$
$\ddot{K}K/\dot{K}^2$	$\tan \alpha [\cos(2\alpha) + (\ddot{\alpha}/2\dot{\alpha}^2)]$	$\frac{1}{2}(1 + (\ddot{B}B/\dot{B}^2))$	$2 + (\ddot{I}I/\dot{I}^2)$

Table 2. Comparison between observations and calculated values for young pulsars

	Crab	PSR B1509-58	PSR B0540-69	Vela
t_p [yr]	915	1550	1660	11000
n_{obs}	2.5179 ± 0.0001	2.837 ± 0.001	2.04 ± 0.02	1.4 ± 0.2
\dot{n}_{obs} [$10^{-4} yr^{-1}$]	-2.7 ± 1.3	-4 ± 11
m_{obs}	10.23 ± 0.03	14.5 ± 3.6
m_{obs} if $\dot{n}_{obs} = 0$	10.160 ± 0.001	13.26 ± 0.01	6.28 ± 0.16	2.8 ± 0.85
P_o [ms]	19	39	25	52
\dot{P}_o [10^{-15}]	559	4760	492	91
$L_o/L_p(I = const)$	6.7	180	8.4	3.7
K_o [$10^{-15} s$]	10.8	186	12.3	4.73
K_o/K_p	0.770	0.802	0.510	0.424
$\frac{K\ddot{K}}{\dot{K}^2}$ ($\dot{n}_{obs} = 0$)	-2.15	-10.27	-0.081	0.737

Fig. 1.— In this diagram, the Crab pulsar track is marked with \times , while PSR B1509-58 is marked with $+$. Each step is $10^3 yr$, from birth to 5×10^4 . Squares indicate the present situation. Both tracks are intercepted by lines representing the same tracks after the conventional model.

Fig. 2.— In this diagram, the Vela pulsar track is marked with x, while PSR B0540-69 is marked with +. Each step is $10^3 yr$, from birth to 5×10^4 . Squares indicate the present situation. Both tracks are intercepted by lines representing the same tracks after the conventional model.